Problem 8

Solve the problem

PDE  

IC  

By using the Fourier transform

Solve:

To solve the PDE by using the Fourier transform, we need first take Fourier transform of the PDE equation and IC equation. Then we can solve the problem that after Fourier transform. At the last we need to take inverse Fourier transform to get real solution.

Step 1 take Fourier transform of PDE and IC.



And then by properties of FT

 (32)

Let,equation (32) becomes

 (33)

Then take Fourier transform of IC:



Step 2 solve the problem under Fourier transform

If we taketo be constant. We could see the problem becomes an ODE problem





The solution of will be



Therefore the solution of ODE is



Step 3 take inverse Fourier Series of the solution.



 (34)

Whichsatisfies

By searching in the table



So 

Since equation (34) will become















Let 

The integral becomes



Take in order to integral.

Integral now becomes



with

After simplified.

The result is

